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INVESTIGATION OF BOUNDARY VALUE PROBLEMS FOR PSEUDOPARABOLIC EQUATIONS AND THEIR APPLICATIONS

ANNOTATION

of the dissertation for the degree of Doctor of Philosophy (PhD)
in the educational program 8D05401- Mathematics

The relevance of the topic.

This dissertation examines nonlocal boundary value problems for third-order linear and nonlinear pseudoparabolic equations.

The theoretical study of boundary value problems with nonlocal conditions for pseudoparabolic equations has developed in connection with their application in solving complex problems in science and engineering. Some of the first results on solving boundary value problems with nonlocal conditions for pseudoparabolic equations were the works of A.F. Chudnovsky, J.R. Cannon, and L.A. Kamynin. Significant contributions to the further development of this area were made by A.M. Nakhushev, V.A. Vodakhova, A.I. Kozhanov, V.Z. Kanchukoev, L.S. Pulkina, V.I. Zhegalov, E.A. Utkina, N.S. Popov, B.S. Ablabekov, K.G. Kozhobekov, Cao Yin J., Jin Ch., N.K. Arkabaev, A.S. Sopuev, U.D. Moldoyarov, Kh.G. Umarov, and other scientists.

In recent years, various analytical and approximate methods for solving pseudoparabolic equations have been actively studied. The relevance of these equations is directly related to their widespread use in the mathematical modeling of physical, technical, and engineering processes. Pseudoparabolic equations are effectively used to describe applied problems such as heat transfer in complex media, filtration processes, the dynamics of deformable bodies, and models of biological reproduction or migration. Therefore, the development and improvement of effective analytical and approximate methods for solving such equations is important not only from a theoretical standpoint but also for practical applications.

This dissertation considers boundary value problems with nonlocal conditions for the Benjamin–Bona–Mahony–Burgers (BBMB) and Benjamin–Bona–Mahony equations. Despite the extensive research devoted to these equations, interest in them remains unabated. This is due to the fact that the BBMB equation is capable of describing a wide class of real-world physical processes: it is used as a model of wave propagation in incompressible fluids, taking into account the simultaneous manifestation of dispersion and dissipation effects, as well as the influence of nonlinearity. Furthermore, the simultaneous presence of dissipative and dispersion terms in the equation makes it a highly complex object from both a mathematical and numerical standpoint. Therefore, the need to develop effective approximate methods for solving applied problems has fueled growing scientific interest in this equation.

The goal of the work. Obtaining conditions for the solvability of non-local boundary value problems for linear and non-linear pseudoparabolic equations of the third order with mixed derivatives, as well as the development of constructive algorithms for finding their solutions.

Research objectives:

- development of an algorithm for finding a solution to a nonlocal boundary value problem for a third-order linear pseudoparabolic equation with a mixed derivative and obtaining conditions for its convergence in terms of the initial data.
- study of the uniqueness of a solution to a nonlocal boundary value problem for one class of linear pseudoparabolic equations.
- based on the algorithm for finding a solution to a nonlocal boundary value problem for a third-order linear pseudoparabolic equation, obtaining conditions for the existence of an "isolated" solution to a nonlocal boundary value problem for a nonlinear pseudoparabolic equation.
- application of the proposed algorithm to the nonlinear Benjamin–Bona–Mahony and Benjamin–Bona–Mahony–Burgers equations and obtaining the necessary conditions for the existence of a solution.

Research methods. The research employs methods of functional analysis and parametrization.

Scientific novelty. For linear and nonlinear pseudoparabolic equations of the third order with nonlocal boundary conditions, the corresponding boundary value problems were investigated and the following results were obtained:

1. An algorithm for solving a nonlocal boundary value problem for a third-order linear pseudoparabolic equation is constructed, and conditions for its convergence are obtained.
2. The uniqueness of a solution to a nonlocal boundary value problem for one class of third-order linear pseudoparabolic equations is investigated.
3. Based on the algorithm for solving a nonlocal boundary value problem for a third-order linear pseudoparabolic equation, conditions for the existence of an "isolated" solution to a nonlocal boundary value problem for a third-order nonlinear pseudoparabolic equation are obtained.
4. The proposed algorithm is applied to the nonlinear Benjamin–Bona–Mahony and Benjamin–Bona–Mahony–Burgers equations, for which conditions for the convergence of the solutions are obtained.

Theoretical and practical significance of the results obtained. The results obtained in the course of the work are of a theoretical nature and can be used in the construction of algorithms for solving non-local boundary value problems for partial differential equations of the third order, as well as in teaching special courses in mathematics in higher educational institutions.

Main results presented for defense:

1. An algorithm for solving a nonlocal boundary value problem for a third-order linear pseudoparabolic equation and conditions for its convergence.
2. Uniqueness of the solution to a nonlocal boundary value problem for one class of third-order linear pseudoparabolic equations.
3. Conditions for the existence of an "isolated" solution to a nonlocal boundary value problem for a third-order nonlinear pseudoparabolic equation, obtained based on an algorithm for solving the corresponding linear problem.
4. Application of the proposed algorithm to the nonlinear Benjamin–Bona–Mahony and Benjamin–Bona–Mahony–Burgers equations and conditions for the convergence of their solutions.

Reliability and validity of the conducted research. The constructive nature

of the methods used in this work ensures the reliability and validity of the results obtained. General conclusions are formulated as theorems and accompanied by proofs.

Approbation of the work. The main results of the dissertation were reported and discussed at the following conferences and seminars:

- International Scientific and Practical Conference «Trends in the Development of Modern Mathematics and Its Teaching in the Context of Digitalization of Education» (April 27–28, 2023, Peoples' Friendship University named after Academician A.Kuatbekov).
- Traditional International April Mathematical Conference (April 5-7, 2023, Institute of Mathematics and Mathematical Modeling, Ministry of Science and Higher Education of the Republic of Kazakhstan, Almaty, Republic of Kazakhstan);
- International Scientific and Practical Conference «Analysis, Differential Equations and Their Applications" dedicated to the 100th anniversary of the birth of T. Y. Amanov, Corresponding Member of the Academy of Sciences of the Kazakh SSR, Doctor of Physical and Mathematical Sciences, Professor (June 22–23, 2023, Astana, Republic of Kazakhstan);
- VII World Congress of Turkic World Mathematicians (September 20–23, 2024, Turkistan, Republic of Kazakhstan);
- Traditional International April Mathematical Conference (April 16-17, 2024, Institute of Mathematics and Mathematical Modeling, Ministry of Science and Higher Education of the Republic of Kazakhstan, Almaty, Republic of Kazakhstan);
- International Scientific Conference «Nonclassical Equations of Mathematical Physics and Their Applications» dedicated to the 90th anniversary of the birth of Academician T. D. Juraev (October 24–26, 2024, National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, Republic of Uzbekistan);
- International Scientific Conference «Actual Problems of Applied Mathematics and Information Technologies – Al-Khwarizmi 2024» (October 22–23, 2024, National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, Republic of Uzbekistan);
- International Summer School and Conference «Evolution Equations, Approximation and Spectral Optimization» (September 11–18, 2024, Institute of Mathematics and Mathematical Modeling, Committee of Science, Ministry of Science and Higher Education of the Republic of Kazakhstan, Almaty, Republic of Kazakhstan).
- International Scientific Conference "Current Issues of Interdisciplinary Scientific Research" dedicated to the 100th anniversary of academician E.A. Buketov (June 17-20, 2025, Karaganda Buketov University, Karaganda, Republic of Kazakhstan).

Publications. The main results of the dissertation have been published in 13 scientific papers and conference proceedings, including 2 papers in journals indexed in Scopus (53rd percentile), 1 paper in a journal indexed in Web of Science (SCIE, Q2), 1 paper in a journal recommended by the authorized body.

The structure and scope of the dissertation. This 92-page dissertation consists of the following structural elements: an introduction, three chapters, a conclusion, and a bibliography.

The first chapter examines a nonlocal boundary value problem with nonlocal conditions for a third-order linear pseudoparabolic equation. By introducing a new variable, the order of the original equation is reduced, after which the time interval is partitioned, yielding a system of equations for the desired functions. An algorithm for finding a corresponding solution is constructed for this system of equations. The convergence of the algorithm is proven, and estimates for the exact and approximate solutions are obtained.

The second chapter, as a generalization of the linear equation considered in the first chapter, examines a nonlinear boundary value problem with nonlocal conditions. This chapter differs from the first in the use of different integral inequalities in the proofs, as well as in the construction of domains in which the existence and uniqueness of the solution are ensured. In the third chapter, solutions to the Benjamin–Bona–Mahony and Benjamin–Bona–Mahony–Burgers equations are obtained using the constructed algorithm. These equations have significant practical significance and describe the application of third-order nonlinear pseudoparabolic equations. The Benjamin–Bona–Mahony and Benjamin–Bona–Mahony–Burgers equations are widely used in physics and applied mathematics to describe wave processes. The Benjamin–Bona–Mahony–Burgers equation finds application in hydrodynamics, plasma physics, elasticity theory, and engineering problems. Thus, the Benjamin–Bona–Mahony equation is primarily oriented toward theoretical research, while the Benjamin–Bona–Mahony–Burgers equation is effectively used to model practical and real-world processes.

Relationship of the Research with Other Scientific Studies:

The dissertation was carried out within the framework of a project funded from the state budget:

AP09259780 “Boundary Value Problems for Pseudoparabolic Equations and the Associated Singular Volterra Integral Equations.”

The topic of the dissertation research, conducted within the field of Natural Sciences, corresponds to the priority area “Intellectual Potential of the Country” and to the specialized scientific direction fundamental and applied research in the fields of mathematics, mechanics, astronomy, physics, chemistry, biology, computer science, and geography.

Contribution of the doctoral candidate to the preparation of each publication. In all published works, the doctoral candidate took an active part in their preparation. In 13 papers written in co-authorship with scientific supervisors and co-authors, the scientific supervisors formulated the problem statement and selected the research methodology, whereas the doctoral candidate independently formulated the main and auxiliary results and provided their proofs.

The number of sources used is 105.

Keywords. Nonlocal boundary value problems, third-order pseudoparabolic equation, parameterization method, conditions for the existence of a solution, algorithm, approximate solution.